



**INDIAN SCHOOL MUSCAT  
HALF YEARLY EXAMINATION  
MATHEMATICS**

CLASS: XI

Sub. Code: 041

Time Allotted: 3 Hrs

29.09.2019

Max. Marks: 80

**General Instructions:**

- (i) All questions are compulsory.
- (ii) Questions in section A are MCQ & very short answer type questions carrying 1 mark each.
- (iii) Questions in section B are short- answer type questions carrying 2 marks each.
- (iv) Questions in section C are long answer - I type questions carrying 4 marks each.
- (v) Questions in section D are long answer - II type questions carrying 6 marks each.

**SECTION - A (Questions 1 to 20 carry 1 mark each).**

**I. CHOOSE THE CORRECT ANSWER. WRITE THE ANSWER ALONG WITH THE CORRECT OPTION: (1x10 = 10 marks)**

1. Find the smallest natural number satisfying the inequality  $20 - 5x < 5(x + 8)$ .  
(a) -1                      (b) 1                      (c) 0                      (d) 2
2. What is the value of  $\frac{\cos 18^\circ - \sin 18^\circ}{\sin 27^\circ}$  ?  
(a)  $\sqrt{2}$                       (b)  $-\sqrt{2}$                       (c)  $\frac{1}{\sqrt{2}}$                       (d)  $\frac{-1}{\sqrt{2}}$
3. If  $n = 10$ ,  $\bar{x} = 12$  and  $\sum x_i^2 = 1530$ , then the coefficient of variation is  
(a) 36                      (b) 41                      (c) 25                      (d) none of these
4. Which of the following is an empty set?  
(a)  $\{x: x \in R, x^2 - 1 = 0\}$                       (b)  $\{x: x \in R, x^2 + 1 = 0\}$   
(c)  $\{x: x \in R, x^2 - 2 = 0\}$                       (d)  $\{x: x \in R, x^2 - x - 2 = 0\}$
5. Let  $R$  be a relation on  $N$  defined by  $x + 2y = 8$ . The domain of  $R$  is :  
(a)  $\{2, 4, 8\}$                       (b)  $\{2, 4, 6, 8\}$                       (c)  $\{2, 4, 6\}$                       (d)  $\{1, 2, 3, 4\}$
6. If  $S_n$  denotes the sum of first  $n$  terms of an A.P whose common difference is  $d$ , then  $S_n - 2S_{n-1} + S_{n-2}$  ( $n > 2$ ) is equal to :  
(a)  $2d$                       (b)  $d$                       (c)  $-d$                       (d) none of these
7. Domain of the function  $f(x) = \frac{1}{\sqrt{x+|x|}}$  is :  
(a)  $[0, \infty)$                       (b)  $(0, \infty)$                       (c)  $(-\infty, \infty)$                       (d)  $(-\infty, 0)$

8. If  $\sum n = 210$ , then the value of  $\sum n^2$  is :  
 (a) 2870 (b) 2160 (c) 2970 (d) 2170
9. If A, B are non-empty sets, then  $(A - B) \cup (B - A)$  equals :  
 (a)  $(A \cup B) - B$  (b)  $A - (A \cap B)$  (c)  $(A \cup B) - (A \cap B)$  (d)  $(A \cup B) \cup (A \cap B)$
10. If  $\cos x = k$  has exactly one solution in  $[0, 2\pi]$ , find the value of  $k$ .  
 (a) -1 (b)  $\frac{1}{\sqrt{2}}$  (c)  $-\frac{1}{2}$  (d) 0

II. **ANSWER THE FOLLOWING : ( 1x10 = 10 marks)**

11. Find the value of  $\tan\left(\frac{23\pi}{6}\right)$
12. Write the radian measure of  $-47^\circ 30'$ .
13. If number of subsets of a set is 32, find the number of elements in the set.
14. Find the range of the function:  $f(x) = 1 - |x - 2|$
15. Find the value of  $9^{1/3} \cdot 9^{1/9} \cdot 9^{1/27} \dots$  upto  $\infty$  terms.
16. Let  $f$  be a function defined by  $f(x) = 5x^2 + 2$ ,  $x \in R$ . Find  $x$  such that  $f(x) = 22$ .
17. If  $A = \{x: x \in N, x < 10\}$  then find the value(s) of  $x$  for which  $-3 < 3x - 1 < x + 5$ .
18. Standard deviation of a collection of data is  $2\sqrt{2}$ . If each observation is multiplied by 3, find the variance of the new data.
19. Write the set  $\left\{\frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \frac{5}{11}, \frac{6}{13}\right\}$  in set-builder form.
20. The marks obtained by Sukriti in first two unit tests are 75 and 70. Find the minimum marks she should get in the third test to have an average of atleast 60 marks.

**SECTION - B (Questions 21 to 26 carry 2 marks each)**

21. If  $A = \{2, 4, 6, 9\}$  and  $B = \{4, 6, 18, 27\}$ , find relation  $R$  defined by  $R = \{(x, y): x \text{ is a factor of } y \text{ and } x < y, x \in A, y \in B\}$ .
22. A horse is tied to a post by a rope 30 m long. If the horse moves along the circumference of a circle always keeping the rope tight, find how far will it have gone when the rope has traced an angle of  $105^\circ$ .
23. Find the mean and variance of first  $n$  natural numbers.  
**OR**  
 Find the mean and variance of first 10 multiples of 3.

24. If  $P = \{x: x < 3, x \in N\}$  and  $Q = \{x: x \leq 2, x \in W\}$ , find  $(P \cup Q) \times (P \cap Q)$ .

25. Find the domain and range of the function  $f(x) = \frac{x+2}{|x+2|}$

OR

Find the domain and range of the function  $f(x) = \sqrt{x-1}$

26. Find the mean deviation about the median for the following data:

$x_i$	10	11	12	13	14
$f_i$	3	12	18	12	3

### SECTION - C (Questions 27 to 32 carry 4 marks each)

27. Prove by the principle of mathematical induction for all  $n \in N$ :

$$1.2.3 + 2.3.4 + 3.4.5 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

28. Given:  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ ,  $A = \{2, 4, 6, 8\}$ ,  $B = \{2, 3, 5, 7\}$ , verify De Morgan's laws.

29. The sum of two numbers is 6 times their geometric mean. Show that the numbers are in the ratio  $(3 + 2\sqrt{2}) : (3 - 2\sqrt{2})$ .

OR

Find the sum to  $n$  terms of the series:  $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \dots$

30. Solve the equation:  $\sin x + \sin 3x + \sin 5x = 0$ .

31. If  $\tan x = \frac{3}{4}$  and  $x$  lies in the third quadrant, find the values of  $\sin \frac{x}{2}$ ,  $\cos \frac{x}{2}$  and  $\tan \frac{x}{2}$ .

OR

Prove that  $\frac{\cos x}{1 - \sin x} = \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$

32. If  $a, b, c$  are in A.P;  $b, c, d$  are in G.P and  $\frac{1}{c}, \frac{1}{d}, \frac{1}{e}$  are in A.P, prove that  $a, c, e$  are in G.P.

### SECTION - D (Questions 33 to 36 carry 6 marks each)

33. In a survey of 25 students, it was found that 15 had taken Mathematics; 12 had taken Physics and 11 had taken Chemistry; 5 had taken Chemistry and Mathematics; 9 had taken Mathematics and Physics; 4 had taken Physics and Chemistry and 3 had taken all the three subjects. Find the number of students who had taken

(i) Physics and Chemistry but not Mathematics

(ii) atleast one of the three subjects

(iii) none of the three subjects.

34. Prove that  $\cos^2 x + \cos^2\left(x + \frac{\pi}{3}\right) + \cos^2\left(x - \frac{\pi}{3}\right) = \frac{3}{2}$

35. Solve the following system of inequalities graphically:

$$3y - 2x \leq 4, \quad x + 3y \geq 3, \quad x + y \leq 5, \quad x \geq 0, y \geq 0.$$

(OR)

A manufacturer has 600 litres of 12% solution of acid. How many litres of 30% acid solution must be added to it so that acid content in the resulting mixture will be more than 15% but less than 18% acid?

36. Calculate the mean, variance and standard deviation for the following data:

Wages ( in Rs)	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of workers	9	17	32	23	40	18	1

OR

For a frequency distribution consisting of 18 observations, the mean and standard deviation were found to be 7 and 4 respectively. But on rechecking, it was found that an observation 12 was miscopied as 21 in calculations. Calculate the correct mean and standard deviation.

**End of the Question Paper**